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A NOVEL APPROACH TO THE ALL- PAIRS SHORTEST PATH PROBLEM IN A DIRECTED Z- GRAPH

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Abstract: The classical Floyd's algorithm is used to find the shortest paths from each vertex to all other vertices. This article focuses on studying the all-pairs shortest path problem in a directed Z-graph. Therefore, a modified version of Floyd's algorithm is applied to find the shortest path between all vertices.

Keywords: Z-number, Directed Z- graph, Floyd's algorithm, Ratio ranking function, Distance matrix and shortest path.

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1. Introduction

Several algorithms, including the Bellman algorithm and the Dijkstra algorithm, have been introduced to resolve the shortest path problem (SPP). These algorithms are designed for graphs with precise edge weights and may not be suitable for those with uncertain edge weights. Several papers have been published on the shortest path problem in a fuzzy environment. Rsanta Kumar raut [6], Asghar Aini [1], Vidhya Kannan [12] and many other researchers have utilized Floyd's algorithm for finding the SPP in a fuzzy environment. The problem of finding the shortest path in a Z- graph has been studied by some researchers [9, 13]. Here, we approach to the all – pairs shortest path in a directed Z- graph using a modified version of Floyd's algorithm.

2. Preliminaries

Definition 2.1: Fuzzy set

A fuzzy set 'A' on a universal set X is a function $A: X \rightarrow [0, 1]$.A is called the membership function, A (x) is called the membership grade of x in A. We write A= {(x, A(x)): x €X}

Definition 2.2: Formal definition of Z-number

A Z-number is an ordered pair of fuzzy numbers (A, B) where A is a fuzzy set defined on the real line and B is a fuzzy number whose support is contained in [0,1].

Definition 2.3: Z-weight of an edge

A Z-graph is a graph with Z-number weighted edges.
The Z-weight of an edge is the Z-distance between the corresponding two nodes.

Definition 2.4: Z-length of the path

Consider a path P from a vertex u to a vertex v. Suppose it consists of edges$e_1, e_2 \dots e_n$. Suppose the weights of these edges are the Z-numbers$(A_1, B_1), (A_2, B_2) \dots (A_n, B_n)$. The Z-length of the path P is denoted by $ZL\ (P)$

$$ZL\ (P) = (A_1, B_1)(+, min)(A_2, B_2) \dots (+, min)\ (A_n, B_n)$$

The shortest path problem in Z- graphs is the problem of finding a path between two specified nodes such that the Z-length of the path is minimum.
Here our aim is to find the path between u and v whose Z-length is minimum. $(Z - SP\ between\ u \&\ v)$.

Definition 2. 5: Operation on fuzzy numbers

Let $A_1 = (a, b, c), A_2 = (d, e, f)$ be two triangular fuzzy numbers.
Then $A_1 + A_2 = (a + d, b + e, c + f)$.

Definition 2.6: Binary operation on Z-numbers

Let (A_1, B_1) and (A_2, B_2) be two Z-numbers. Then
$(A_1, B_1)(+, min)(A_2, B_2) = (A_1 + A_2, \min(B_1, B_2))$

Definition 2.7: Min R Operation [11]

Let $*$ be any one of the basic arithmetic operations addition, subtraction, multiplication or division. Let R_k be any suitably chosen ranking function. Then the Min R Operation is defined by $(A, B)(*, min)(C, D) = (A * C, min(B, D))$, where $A * C$ is calculated by using the extension principle, $min(B, D) = B\ if\ R_k(B) \le R_k(D)\ and\ \ D\ if\ R_k(D) < R_k(B)$

Definition 2.8: Distance matrix of a directed graph

Let D be a directed graph with p vertices. The distance matrix is the pxp matrix whose $(i, j)^{th}$ entry gives the distance from the point v_i to the point v_j and ∞ if there is no path from $v_i\ to\ v_j$.

Definition 2.9: Distance matrix of Z- graph

The distance matrix of Z- graph (V, E) is a nxn matrix D of edge weights in the directed graph.

$$D\ (i, j) = \begin{cases} ((0,0,0),(1,1,1)) & if\ \ i = j \\ \text{Weight of the edge from i to j} & if\ i \ne j\ and\ (i, j) \in E \\ ((\infty,\infty,\infty),(1,1,1)) & if\ i \ne j\ and\ (i, j) \notin E \end{cases}$$

3. All – pairs shortest path problem in a Z-graph

Let $G = (V, E)$ be a directed Z-graph with positive edge weight. Given any two vertices $u, v,$ we would like to know the shortest path. Floyd algorithm is used to compute the minimum path between every pair of vertices.

For given a path $P = \{v_1, v_2 \dots v_n\}$, the vertices $v_2, \dots, v_{n-1}$ are referred to as the intermediate vertices of this path. Define $dij^{(k)}$ as the shortest path from i to j such that any intermediate vertices on the path are chosen from the set $\{1, \dots, k\}$.

For the base case where $k = 0$, the path cannot include any intermediate vertices, and so

$d_{ij}^{(0)} = weight\ of\ the\ edge\ from\ i\ to\ j\ for\ all\ i, j.$ (ie) a path consisting of a single edge has no intermediate vertices.

Moving on to the induction step $(k \geq 1)$, there are two cases while updating d_{ij}:

For finding the shortest path, we assume that intermediate vertices are chosen from $\{1, 2 \ldots k\}$.

In the first case, the list of its intermediate vertices doesn't contain the k[th] vertex.

The shortest path from vertex i to vertex j involves only the intermediate vertices $\{1, 2 \ldots k - 1\}$ and hence $d_{ij}^{(k)} = d_{ij}^{(k-1)}$

In the second case, the path contains the k^{th} vertex among the intermediate vertices. Without loss of generality, we assume that the k_{th} vertex occur only once in the list, and hence the length of the shortest path $is\ d_{ij}^{(k)} = d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$

We use the formula $d_{ij}^{(k)} = min\{d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\}$

The output will be a nxn distance matrix $D = (d_{ij})$, where d_{ij} - the shortest path length from vertex $i\ to\ j$.

Also, it is a minimization problem. So, we are interested in minimizing the path length given by Z-number (A, B). We need to minimize the first component A which represents the path length but maximize the second component B which refers to the reliability. Since the second component is non negative it is enough to minimize the reciprocal of the second component.

Hence we use the ratio ranking function $\mathrm{RA}((\mathrm{A}, \mathrm{B})) = \frac{R_1(A)}{R_2(B)}$, where $R_1\ and\ R_2$ are ranking function on set of fuzzy numbers.

Hence, we use Ratio ranking method for comparing Z- numbers and for addition, $Min\ R$ operation is applied.

3.1 Modified form of Floyd's algorithm

Inputs:

$V = \{1,2,3\ldots\ldots n\}$ // the set of vertices in the given connected directed Z-graph G

$d_{ij}\ for\ i, j = 1,2, \ldots n$// $d_{ij} = (d_{ij1}, d_{ij2}) = length\ of\ arc(i, j)\ in\ Z - number\ format$

Initialisation:

$P\ (i, j, 0) = P\ (i, j)\ \forall\ i, j = 1 to\ n$

$For\ i = 1\ to\ n$

{

$For\ j = 1\ to\ n$

{

$d_{ij}^{(0)} = d_{ij}$

}

}

Main part:

$For\ k = 1\ to\ n$

{
$For\ i = 1\ to\ n$
{
$For\ j = 1\ to\ n$
{
If $R\,(d_{ij}^{(k-1)}) \leq R\,(d_{ik}^{(k-1)} + d_{kj}^{(k-1)})$ $//\ Here\ +\ indicates\ (+, min)\ operation$

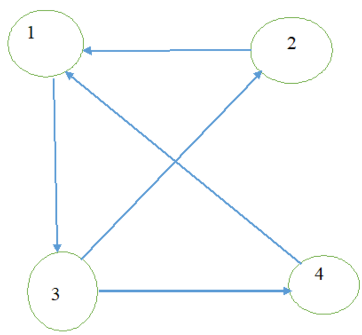
$d_{ij}^{(k)} = d_{ij}^{(k-1)}\ and\ P(i,j,k) = P(i,j,k-1)$
else
$d_{ij}^{(k)} = d_{ik}^{(k-1)} + d_{kj}^{(k-1)}$ and P(i, j, k) =P(i, k,k-1) ∪ P(k, j,k-1)
}
}
}

Output:

- The distance matrix of shortest paths' lengths
- The matrix containing the shortest paths of every vertex.

3.2 Example:

Consider the directed Z-graph G in figure 1.



$Figure\ 1$

Here, $V = \{1, 2, 3, 4\}$

The weight of the edges of G are given in the table1

Edge	Z-Weight
21	((10,15,20), (.8, .85, .9))
13	((20,25,30), (.7, .75, .8))
32	((30,35,40), (.9, .95, 1))
34	((50,55,60), (.5, .55, .6))
41	((70,75,80), (.6, .65, .7))

$Table\ 1$

Now, our objective is to find the shortest path from every vertex of G.

$D^{(0)} =$

$$\begin{bmatrix} ((0,0,0),(1,1,1)) & ((\infty,\infty,\infty),(1,1,1)) & ((20,25,30),(.7,.75,.8)) & ((\infty,\infty,\infty),(1,1,1)) \\ ((10,15,20),(.8,.85,.9)) & ((0,0,0),(1,1,1)) & ((\infty,\infty,\infty),(1,1,1)) & ((\infty,\infty,\infty),(1,1,1)) \\ ((\infty,\infty,\infty),(1,1,1)) & ((30,35,40),(.9,.95,1)) & ((0,0,0),(1,1,1)) & ((50,55,60),(.5,.55,.6)) \\ ((70,75,80),(.6,.65,.7)) & ((\infty,\infty,\infty),(1,1,1)) & ((\infty,\infty,\infty),(1,1,1)) & ((0,0,0),(1,1,1)) \end{bmatrix}$$

$\boldsymbol{Step\ 1}$: $for\ k = 1$

$R\,(d_{11}^{(0)}) = 0\ and\ R\,(d_{11}^{((0))} + d_{11}^{(0)}) = 0$

Here, $R\,(d_{11}^{((0))}) = R(d_{11}^{((0))} + d_{11}^{(0)})$

□ $d_{11}^{((1))} = d_{11}^{(0)}$ $(by\ if\ condition)$

Now, $R\,(d_{12}^{(0)}) = \infty\ and\ R\,(d_{11}^{(0)} + d_{12}^{(0)}) = \infty$

Here, $R\,(d_{12}^{(0)}) = R(d_{11}^{(0)} + d_{12}^{(0)})$

□ $d_{12}^{(1)} = d_{12}^{(0)}$ $(by\ if\ condition)$

$R\,(d_{23}^{(0)}) = \infty\ and$

$R\,(d_{21}^{(0)} + d_{13}^{(0)}) = R\,\{((10,15,20),(.8,.85,.9)) + ((20,25,30),(.7,.75,.8))\}$

$$= R\,\{((30,40,50),(.7,.75,.8))\} = \frac{R_1(30,40,50)}{R_2(.7,.75,.8)} = 12.84$$

Here, $R\,(d_{23}^{(0)}) > R(d_{21}^{(0)} + d_{13}^{(0)})$

□ $d_{23}^{(1)} = d_{21}^{(0)} + d_{13}^{(0)}$ $(by\ else\ condition)$

In a similar manner, we get

D $^{(1)}$ =

$$\begin{bmatrix} ((0,0,0),(1,1,1)) & ((\infty,\infty,\infty),(1,1,1)) & ((20,25,30),(.7,.75,.8)) & ((\infty,\infty,\infty),(1,1,1)) \\ ((10,15,20),(.8,.85,.9)) & ((0,0,0),(1,1,1)) & ((30,40,50),(.7,.75,.8)) & ((\infty,\infty,\infty),(1,1,1)) \\ ((\infty,\infty,\infty),(1,1,1)) & ((30,35,40),(.9,.95,1)) & ((0,0,0),(1,1,1)) & ((50,55,60),(.5,.55,.6)) \\ ((70,75,80),(.6,.65,.7)) & ((\infty,\infty,\infty),(1,1,1)) & ((90,100,110),(.6,.65,.7)) & ((0,0,0),(1,1,1)) \end{bmatrix}$$

We use $P\,(i,j,k) = P\,(i,k,k-1)\,U\,P\,(k,j,k-1)$, for finding the length of the path

$P\,(1,1,1) = P\,(1,1,0)\,U\,P\,(1,1,0) = ((0,0,0),(1,1,1))$

$P\,(1,2,1) = P\,(1,1,0)\,U\,P\,(1,2,0) = P\,(1,2)$

In a similar way, we get

$$P^{(1)} = \begin{pmatrix} 1 \to 1 & 1 \to 2 & 1 \to 3 & 1 \to 4 \\ 2 \to 1 & 2 \to 2 & 2 \to 1 \to 3 & 2 \to 4 \\ 3 \to 1 & 3 \to 2 & 3 \to 3 & 3 \to 4 \\ 4 \to 1 & 4 \to 2 & 4 \to 1 \to 3 & 4 \to 4 \end{pmatrix}$$

$\boldsymbol{Step\ 2}$: $for\ k = 2$

$$D^{(2)} = \begin{bmatrix} ((0,0,0),(1,1,1)) & ((\infty,\infty,\infty),(1,1,1)) & ((20,25,30),(.7,.75,.8)) & ((\infty,\infty,\infty),(1,1,1)) \\ ((10,15,20),(.8,.85,.9)) & ((0,0,0),(1,1,1)) & \big((30,40,50),(.7,.75,.8)\big) & ((\infty,\infty,\infty),(1,1,1)) \\ ((40,50,60),((.8,.85,.9)) & ((30,35,40),(.9,.95,1)) & ((0,0,0),(1,1,1)) & ((50,55,60),(.5,.55,.6)) \\ ((70,75,80),(.6,.65,.7)) & ((\infty,\infty,\infty),(1,1,1)) & ((90,100,110),(.6,.65,.7)) & ((0,0,0),(1,1,1)) \end{bmatrix}$$

and
$$P^{(2)} = \begin{pmatrix} 1 \to 1 & 1 \to 2 & 1 \to 3 & 1 \to 4 \\ 2 \to 1 & 2 \to 2 & 2 \to 1 \to 3 & 2 \to 4 \\ 3 \to 2 \to 1 & 3 \to 2 & 3 \to 3 & 3 \to 4 \\ 4 \to 1 & 4 \to 2 & 4 \to 1 \to 3 & 4 \to 4 \end{pmatrix}$$

$\boldsymbol{Step\ 3}$: $For\ k = 3$

$$D^{(3)} = \begin{bmatrix} ((0,0,0),(1,1,1)) & (50,60,70),(.7,.75,.8))) & ((20,25,30),(.7,.75,.8)) & ((70,80,90),(.5,.55,.6)) \\ ((10,15,20),(.8,.85,.9)) & ((0,0,0),(1,1,1)) & ((30,40,50),(.7,.75,.8)) & ((80,95,110),(.5,.55,.6)) \\ ((40,50,60),(.8,.85,.9)) & ((30,35,40),(.9,.95,1)) & ((0,0,0),(1,1,1)) & ((50,55,60),(.5,.55,.6)) \\ ((70,75,80),(.6,.65,.7)) & ((120,135,150),(.6,.65,.7)) & ((90,100,110),(.6,.65,.7)) & ((0,0,0),(1,1,1)) \end{bmatrix}$$

and
$$P^{(3)} = \begin{pmatrix} 1 \to 1 & 1 \to 3 \to 2 & 1 \to 3 & 1 \to 3 \to 4 \\ 2 \to 1 & 2 \to 2 & 2 \to 1 \to 3 & 2 \to 3 \to 4 \\ 3 \to 2 \to 1 & 3 \to 2 & 3 \to 3 & 3 \to 4 \\ 4 \to 1 & 4 \to 3 \to 2 & 4 \to 1 \to 3 & 4 \to 4 \end{pmatrix}$$

$\boldsymbol{Step\ 4}$: $for\ k = 4$

$$D^{(4)} = \begin{bmatrix} ((0,0,0),(1,1,1)) & (50,60,70),(.7,.75,.8))) & ((20,25,30),(.7,.75,.8)) & ((70,80,90),(.5,.55,.6)) \\ ((10,15,20),(.8,.85,.9)) & ((0,0,0),(1,1,1)) & ((30,40,50),(.7,.75,.8)) & ((80,95,110),(.5,.55,.6)) \\ ((40,50,60),(.8,.85,.9)) & ((30,35,40),(.9,.95,1)) & ((0,0,0),(1,1,1)) & ((50,55,60),(.5,.55,.6)) \\ ((70,75,80),(.6,.65,.7)) & ((120,135,150),(.6,.65,.7)) & ((90,100,110),(.6,.65,.7)) & ((0,0,0),(1,1,1)) \end{bmatrix}$$

and
$$P^{(4)} = \begin{pmatrix} 1 \to 1 & 1 \to 3 \to 2 & 1 \to 3 & 1 \to 3 \to 4 \\ 2 \to 1 & 2 \to 2 & 2 \to 1 \to 3 & 2 \to 3 \to 4 \\ 3 \to 2 \to 1 & 3 \to 2 & 3 \to 3 & 3 \to 4 \\ 4 \to 1 & 4 \to 3 \to 2 & 4 \to 1 \to 3 & 4 \to 4 \end{pmatrix}$$

Hence, the shortest path and its distance of a directed Z-graph from every vertex to other vertices is obtained.

4. Conclusion

In this paper, we have made suitable adjustments to the well- known Floyd's algorithm, to apply it to a directed Z-graph. This allows us to find the shortest path between every pair of vertices in the Z-graph and obtain the Z-length of the shortest path.

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